

SPE-173278-MS

Hybrid Optimization for Closed-Loop Reservoir Management

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This paper was prepared for presentation at the SPE Reservoir Simulation Symposium held in Houston, Texas, USA, 23-25 February 2015.

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Abstract

The closed-loop optimization paradigm of an oil field can increase oil recovery and reduce water production, maximizing economic gains. One way to improve the management of a field involves designing optimal production strategies by means of dynamically adjusting the production flow rates or bottom-hole pressures over the reservoir life-cycle and operation. A major difficulty occurs in optimizing production of all wells, according to constraint along the production of a field. These optimal control strategies are often difficult to be realized in practice due to the large number of control variables to be adjusted during the optimization process, requiring large amount of computational infrastructure in place. These challenges become even more evident with larger number of wells and with complex large-scale reservoirs. For these reasons, this work proposes a new hierarchical hybrid optimization framework employing model order reduction techniques in a closed-loop fashion. This paper proposes the use of proper orthogonal decomposition (POD) with the discrete empirical interpolation method (DEIM), to reduce the computational effort, and to perform local optimization by means of gradient-based approach by using forward and adjoint models followed by aggressive line search process. This approach was applied in the UNISIM-I-D benchmark case, testing the performance of the optimization proposed in a complex reservoir with several producer and injector wells, whose conventional optimization would require a high computational cost. The results showed an improvement in reservoir management by means of additional gains in terms of NPV, and through the proposed robust optimization algorithm, we show advantages of the operation of the wells and in the reduction in the computational efforts necessary to attain optimal solutions. The efficiency of the gradient-based approach coupled with model order reduction can be combined in future entire optimization workflow with global optimum algorithms like Fast Genetic Algorithm.

Introduction

The development of an oil field involves many challenges, among them: (1) maximize profit and reduced risks, (2) achieve higher oil recovery, (3) reduce uncertainties associated with reservoir characteristics (static and dynamic properties), and (4) decrease the high flow of water as a limiting factor in the oil production. Although the weight put into some of these issues depend upon the life stage of an oil field,



Figure 1—UNISIM-I-D benchmark case used in this article.

and can be non-existent in many circumstances, the objective of any project development in certainly related to profits and risks.

The management of an oil field can be tackled by finding the optimal production strategy (or strategies during the reservoir life cycle) by means of dynamically adjusting the production flow rates or bottom-hole pressures over the reservoir production time. This optimization, in turn, can be tight to some form of economical objective function, very often indicated by the net present value (NPV) of the operations. As it is known, realistic field production is not set as the operator will, but it is driven by operations constraints defined by the capacity of the

Table	1	Econom	ic	Scena	ria
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Parameter	Value	Unity	
Oil price	62.9	USD/m ³	
Water production cost	6.29	USD/m ³	
Water injection cost	6.29	USD/m ³	

Table 2—Bounded restrictions of the wells for the optimization process

Producer	Wells	Injector W	Vells
Minimum BHP	100 barsa	Minimum BHP	300 barsa
Maximum BHP	300 barsa	Maximum BHP	700 barsa

platform and the surface facilities required to separate, process and store (or drain) all production (oil, water and gas). A major difficulty occurs in optimizing production of all wells, according to this constraint along the production of a field. For this reason, designing optimal well control strategies taking into account realistic production constraints and well allocation rates is of paramount for the increase in reservoir recovery factor and, in turn, the NPV of the project. Accordingly, this article aims to develop a methodology for the optimization of the flow of the wells over life-cycle time to produce a field.

Many studies published in the literature (Litvak et al., 2002; Brouwer et al., 2004; Liang et al., 2007; van Essen et al., 2009; Cardoso & Durlofsky, 2010; Viadana et al., 2012; Gildin et al., 2013) emphasize the need and importance of performing closed-loop optimization using optimal control theory. These proposed algorithms have been implemented in benchmarks developed for the optimization under the uncertain paradigm, e.g. Norne and Brugge fields (Rwechungura et al., 2010; Peters et al 2013).

The overall experience is that the methodologies implemented can indeed improve the economic gains and reduced uncertainties in the field. However, they rely on large-scale computations owing to the fine-scale reservoir models derived for the full field simulations. In order to overcome this difficult, this work proposes a new hierarchical hybrid optimization framework employing model order reduction techniques in a closed-loop fashion. We utilize the new reservoir benchmark called UNISIM-I-D (Gaspar et al., 2013).

Reduced-order modeling techniques have shown to be a viable way of mitigating the large-scale nature of the simulation models and accelerate the computations taking place in subsurface applications. The options range from non-intrusive methods, i.e., do not depend on modifications of a reservoir simulation code, to a more intrusive and sophisticated methods that depend on several modifications of legacy code or the development of new simulator codes. For the intrusive schemes, reduced-order modeling by projection has been used in the systems/controls-like framework, such as the balanced truncation and proper orthogonal decompositions (POD) (Volkwein & Hinze, 2005; Gildin et al., 2013), trajectorypiecewise linear (TPWL) (Cardoso & Durlofsky, 2010), bilinear Krylov subspace methods (Ghasemi et al., 2014) and quadratic bilinear model order reduction (Gildin and Ghasemi, 2014). In Chaturantabut et al. (2010), the authors utilized a variant of POD for nonlinear systems, called POD-DEIM (discrete empirical interpolation method) in which the nonlinear terms are approximated by some form of interpolation, and therefore great reductions can be achieved in computational effort.

As far as the optimization, we employ a local optimization, which can be combined in future work with a global optimization workflow. On a higher level, one can employ a fast genetic algorithm (FGA), which is a robust and efficient method for sweeping the solution space with many variables and it has been used in some studies (Almeida et al., 2010; Sampaio et al., 2011; Sampaio et al., 2012), showing significant progress compared to classical



Figure 2—Schematic of POD-DEIM algorithm

optimization methods. Due to its low efficiency in computational time and in finding the local maximum, we introduce a lower level optimization in our framework. On a lower level, the local optimization was performed by gradient-based method for a better refinement of the local solution. This approach takes advantage of the gradient-based approaches for local maximization, but at the same time, the computations are more efficiently than using GA for the entire optimization workflow.

In order to simplify our framework, we consider deterministic conditions, i.e., there is no need to realize the history matching of the model. To this end, the production optimization strategies are defined after the reservoir is being development, i.e., after the project variables, namely, the number of wells, type, placement and schedule, and capacities constraints of liquid, oil and water production and also water injection are pre-defined.

This manuscript is organized as follows. We start by introducing the UNISIM-I benchmark model and state the simplifications taken in this paper. We then review model reduction and optimization and finally apply these concepts to the simplified UNISIM. Some discussions and conclusions are drawn in the end of the paper.

UNISIM-I-D Benchmark

Reservoir Model

The UNISIM-I was developed in response to the Brazilian campaigns in dealing with carbonate reservoirs. The reference model is based (after some modifications) on the structural, facies and petrophysical characteristics of the Namorado Field, located in Campos Basin, Brazil. The original model as described in (Gaspar et al., 2013), contains approximately 3.5 million of active grid blocks. In this paper, we use a



Figure 3—Flowchart of the local optimization framework.





modified version of the original reservoir, such that the dimensions of the grid are 81x58x20 blocks with approximately 37,000 active cells. The grid used in the models is Corner-point. The optimization methodology was applied to the UNISIM-I-D benchmark case (Gaspar et al., 2013) to optimize the control variables in an optimized production strategy. Figure 1 show the model used in this work with final oil saturation.

Geological, PVT and petrophysical data for this model are available (Gaspar et al., 2013). The production history data contains liquid production rates of producer wells. Core description, well log information and seismic data were used to build the structural, facies and petrophysical models.

Economic Scenario

We used a very simple economic scenario as shown in Table 1.

Well Configurations

Fourteen producers and eleven injector wells were used, all horizontal with 500 m of length. The producer wells were completed in the second layer of the model and the injectors, the tenth and last layer of the model. For both injectors and producers, the maximum and minimum bottomhole pressure were defined as upper and lower boundaries for the optimization process, as listed in Table 2.

Methodology

Step 1: Model Order Reduction – POD-DEIM

The proper orthogonal decomposition (POD) method has been used in several disciplines, ranging from aerodynamics, to thermal systems to finance, and to weather prediction (see Antoulas, et al. 2001). The



	# Pressure	# Velocity	# Face Pressure	# Saturation	Simulation Time (s)	
Fine Scale Model	38466	120897	9499	38466	259	
Reduced Model	5	23	5	18	64	

method is simple to implement, as the projection matrices can be computed from several forward simulations of a training set of inputs and the recorded snapshots of the solution variables. The POD method has been applied in the reservoir simulation and production optimization (Brouwer et al., 2004). Gildin et al. (2013) and Ghasemi et al., (2015) proposed the use of proper orthogonal decomposition (POD) to perform the reduction of reservoir model, and thus reduce the computation time spent in the simulations. To reduce the complexity of the nonlinear terms and increase the speed of each run, the POD was modified by adding a discrete empirical interpolation method (DEIM), forming the POD-DEIM method. Here we briefly explain the application of POD and its modification based on DEIM.

The first step is the training phase in which snapshots of high fidelity model at each time step, the pressure, saturation, velocity, etc., of all grid blocks are saved. These snapshots are assembled into snapshot matrices. After applying a singular value decomposition (SVD) on these matrices, we select the projection matrix by indicating the fraction of total energy to be captured. The fractional energy is defined as,

$$E = \frac{\sum_{i=1}^{l} \sigma_i}{\sum_{i=1}^{n_s} \sigma_i} \tag{1}$$



Figure 6—Relative saturation error between reduced model and high fidelity one



Figure 7—Comparison of water cut at producers between reduced model (solid line) and high fidelity one (dashed line)



Figure 8—Optimal schedule using high fidelity model.

The number of basis, l, is selected such that 0.9 < E < 1 to have reasonable error. After finding number of basis from each matrix of solutions, the corresponding basis matrix Φ can be constructed by the first l columns of the corresponding eigenvector matrix. Therefore, the new state variable can be defined as,

$$\mathbf{x} = \Phi \mathbf{x}_r \tag{2}$$

where x is any state of the system and x_r is the reduced state. Substituting the new reduced state into the pressure and saturation equation results in reduced systems as follows,

$$\begin{pmatrix} \phi_v^T B \phi_v & \phi_v^T C \phi_p \\ -\phi_p^T C^T \phi_v & 0 \end{pmatrix} \begin{pmatrix} v_r \\ p_r \end{pmatrix} = \begin{pmatrix} 0 \\ f \end{pmatrix}$$

$$\mathcal{R}_r(s_r) \triangleq \phi_s^T \mathcal{R}(\phi_s \, s_r, s^n, u^{n+1})$$
(3)
(3)
(3)

where v_r , p_r and s_r are reduced flux, pressure and saturation, respectively. The reduced pressure equation in (3) is a determined linear system and can be solved by direct solver or iterative solvers efficiently. The saturation equation results in a reduced residual equation as in (4). This equation is nonlinear and can be solved efficiently by using the Newton-Raphson method. Note that the corresponding Jacobian of the reduced residual defined as,

$$\mathcal{J}_r \triangleq \frac{\partial \mathcal{R}_r}{\partial \mathbf{s}_r} = \phi^T \frac{\partial \mathcal{R}}{\partial \mathbf{s}_r} = \phi^T \frac{\partial \mathcal{R}}{\partial s} \frac{\partial s}{\partial s_r} = \phi^T \mathcal{J}\phi$$
⁽⁵⁾

In general, implementing POD in the reservoir simulator is fairly easy as long as the source code is available. The main drawback of the POD stems from the fact that at each iteration the resulted results should be projected back to the full scale solution to update the fluid properties and find the full residual and Jacobian, which are nonlinear functions of state variable. However, one can modify the POD method to reduce the complexity of the nonlinear terms. One of these methods is discrete empirical interpolation (DEIM) as will be explained in the next section.

POD-DEIM The Discrete Empirical Interpolation Method (DEIM) is a variant of the Empirical Interpolation Method (EIM) proposed for reducing the complexity of the solution of partial differential equations (PDE) by replacing the orthogonal projection of POD-Galerkin with an oblique projection. In



Figure 9-Optimal schedule using reduced model.

this case, the nonlinearities are also approximated based on interpolation of a POD-like basis obtained from saving a snapshot matrix formed by the nonlinear terms. It constructs an interpolation function used for the evaluation of the nonlinear terms, by selecting an optimal subset of indexes through a greedy algorithm and the nonlinear function only needs to be evaluated at this greatly reduced set of component.

Mathematically one can approximate the nonlinear function $g(\mathbf{x})$ by projecting it onto the subspace spanned by a basis $\{\Psi_1, \ldots, \Psi_m\} \subset \mathbb{R}^N$ of dimension m < N as,

$$g(x) \simeq \sum_{i=1}^{m} c_i(x)\psi_i = \psi c(x)$$
(6)

where the columns of Ψ are the projection basis obtained by applying *svd* to the nonlinear function snapshots, and c(x) are the corresponding coefficient vector. These coefficient are determined by selecting the *m* rows of the overdetermined system in Eq. 14 that span the largest range of solutions, through greedy algorithm (Chaturantabut et al., 2010), as follows

$$c = (P^T \psi_m)^{-1} P^T \mathbf{g}(x) \tag{7}$$

where Ψ_m is the first *m* column of matrix Ψ and *P* is a selection matrix defined as,

$$P = [\mathbf{e}_{p1}, \dots, \mathbf{e}_{pm}] \tag{8}$$

where $\mathbf{e}p_1$ is a vector that is zero everywhere except index p_1 and so on.

In reservoir simulation the rock and fluid properties are nonlinear functions of pressure or saturation. Also, depending on the problem formulation, one may deal with mobility or fractional fluid as a nonlinear function of pressure and saturation. Evaluating these functions (and also the derivatives) at all cells is computationally expensive. Specifically in the case of applying POD, it requires projecting back to fine scale model at each iteration. Therefore, we apply DEIM to evaluate these functions at very few cells and find the properties at the rest of the cells by interpolation. The framework of POD-DEIM is illustrated in Figure 3.

POD-DEIM Algorithm Run the fine scale model with some training inputs (variation of inputs in their possible range) and save the pressure, saturation and the nonlinear function at each time step;

Select the appropriate number of basis based on *svd* to capture the necessary energy of each variable;

Apply *deim* function to nonlinear function basis to find the most important cells (selection matrix) based on greedy algorithm;

Approximate all the nonlinear functions in formulation with DEIM.



Figure 10-Top view of final oil saturation with optimal schedule after 1500 days.

Step 2: Local Optimization Process

This local optimization comprises forward model and adjoint model, which are combined to find the gradient, followed by aggressive line search, which finds the greatest growth of the function in the gradient direction. In future work, this local optimization process can follow by global optimization process like Fast Genetic Algorithm with advanced genetic operators that aims to accelerate the search for the global maximum. Figure 3 describes the implementations of this method.

Case Study

This section presents the application of the methodology in a reservoir model based on a Brazilian offshore oil field, see (Gaspar et al., 2013). This model was ran in MATLAB Reservoir Simulation Toolbox (MRST, see Aarnes et al. (2007)).

Results and Discussions

In this section we apply the optimization on waterflooding the reservoir. It is assume that the reservoir is above bubble point pressure and the reservoir is saturated with oil and water. Note that we assumed the reservoir is saturated with oil at the initial time. We also assumed Corey's model for the relative permeability curve as a quadratic function. The permeability of the reservoir is as provide in the benchmark.

Model Order Reduction

Here we apply the model order reduction work flow explained in this paper to the UNISIM benchmark. The model was run for 1000 days based on schedules shown in Figure 4. for bottom-hole pressure of the producers and injectors. The pressure (Po), flux (Vel), water saturation (Sw) and fractional flow of water (fw) are saved every 10 days. Thus we have 100 snapshots for each state. After applying *svd* to each snapshot matrix, the singular values are obtained and are compared in Figure 5. As can be seen, there is

a faster decay in the singular values for the pressure and velocity compared to saturation and fractional function. Thus, we need more basis for saturation and nonlinear functions of it to have small error.

The selection criteria here was to capture at least 99% of the energy of snapshots. The number of basis is compared between the reduced model and the original fine scale one in Table 3. It is obvious that several orders of magnitude in model order reduction is obtained in this example. Also, the simulation runtime reduced more than 4 time.

The relative saturation error at each time step is calculated as follows,

$$RelErrSat(t_i) = \left\| s_{ref}(t_i) - \phi_s s_r(t_i) \right\| / \left\| s_{ref}(t_i) \right\|.$$

This error is shown in Figure 6 for this model. As it is shown the error is less than 2% for most of the simulation time. Figure 7 compares the water cut between the reduced model and the high fidelity reservoir model. It reveals that the reduced model can reproduce very similar results with less computational runtime.

Optimization Process

In this section, we run gradient based optimization to find the bottom-hole pressure for 14 producers and 11 injectors in the field for a period of 1500 days. It is assumed that the bottom-hole pressure can be changed every 30 days. Thus, we have 1250 control variables. This problem can be very computationally expensive to run considering at each iteration the reservoir model has to be run.

The results of this optimization is compared in Table 4, which reveals that the reduced model resulted in a close NPV, but with much less computational effort. As can be seen, the number of iterations is reduced by one order of magnitude as the final runtime. Although we have not attined an NPV compared with the fine scale model, we belive this can be fixed by making a judicioius choice on the number of basis of the reduced model. Also, we ran the model for only 5 years of production and a better measure of the dynamics of the reservoir would be attained with a longer time span.

The optimal bottom-hole pressure for producers and injectors are shown in Figure 8 and Figure 9 for high fidelity model and reduced model, respectively. The final oil saturation in the reservoir after 1500 days by applying the optimal schedule is shown in Figure 10. As can be seen, the reduced order model was able to work as a surrogate model inside the optimization. The oil saturation shows an efficient migration of the oil into the producers.

Conclusions

In this paper we implemented optimization algorithms based on a reduced-order model for the UNISIM benchmark. Model reduction was implemented to overcome the high computational cost associated with the gradient-based optimizations. As it was pointed out, we should expect to gain accuracy and computational savings by employing the hybrid approach, where the optimal well controls can be obtained by a hierarchical framework using FGA on a higher level.

Nomenclature

- μ = viscosity, cp
- $\varphi = \text{porosity}$
- q = flow rate, bbl/day
- x = state variable vector
- R = residual
- J = Jacobian
- N = number of grid blocks
- 1 =number of POD basis
- m = number of DEIM basis

t = time

Subscripts

- p =pressure
- s =water saturation
- r =reduced
- o =relative to oil
- w =relative to water

Subscripts

- n =time step number
- k =iteration number

Acknowledgments

This paper received financial support from FAPESP (grant number: 2014/07917-0). The authors would like to thank Petrobras, UNISIM, CEPETRO, UNICAMP, Texas A&M University and Santa Catarina State University for supporting this research and development project. This paper was also funded in part by the US Army Research Office, Contract No. W911NF-12-1-0206 and FCMG Research Chair at TexasA&M.

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